
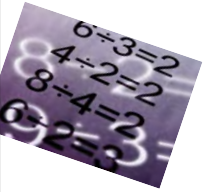


Critical Foundations for Algebra



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Group Norms

Listening: SLANT

Cell phone reminder

Conversations

Breaks




Bathroom location

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Response Cards

- Purpose
- Critical Foundations for Algebra poll



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National Mathematics Advisory Panel Report



Doing What Works
Resources are based
on the
recommendations of
the National
Mathematics Advisory
Panel



The National Mathematics Advisory Panel conducted a systematic and rigorous review of the best available scientific evidence for the teaching and learning of mathematics and provided recommendations that lay out concrete steps to improve mathematics education, with a specific focus on preparation for learning algebra.

Critical Foundations for Algebra

The National Mathematics Advisory Panel report offers recommendations for how to best prepare elementary and middle school students for success in algebra, a gateway to mathematics for high school and beyond.

Mathematics Preparation for Algebra

- Follow a focused, coherent progression of mathematics
- Achieve proficiency with whole numbers, fractions, and aspects of geometry and measurement
- Build deep understanding
- Emphasize fractions and related concepts

Comprehensive Instruction

- Develop conceptual understanding, computational fluency, and problem-solving skills
- Achieve automaticity in computation
- Provide adaptive practice
- Encourage effort and persistence

Mastery Framework

- Set benchmarks for key skills
- Use formative assessments
- Provide explicit instruction for struggling students
- Offer acceleration and enrichment for gifted students

Grade-Level Benchmarks for Critical Foundations¹

5th	Add and subtract whole numbers
6th	Identify, represent, and compare fractions and decimals
7th	Multiply and divide whole numbers
8th	Compare, add, and subtract fractions and decimals
9th	Solve problems with perimeter and area
10th	Multiply and divide fractions and decimals
11th	Use all operations on integers
12th	Analyze properties and measures with 2- and 3-D shapes
13th	Use all operations on positive and negative fractions
14th	Solve problems with percent, rate, and proportion
15th	Relate similar triangles with slope of a line

¹ For full report, including text for the benchmarks, please see: www.ncgi.org/ncgi/education/ncgi/ncgi-panel-report/ncgi-report.pdf

Module Overview





National Mathematics Advisory Panel




It's not just in Montana

Bradley Witzel from the Montana Reading Institute 2010

Montana educators identify by grade level topics students struggle with.

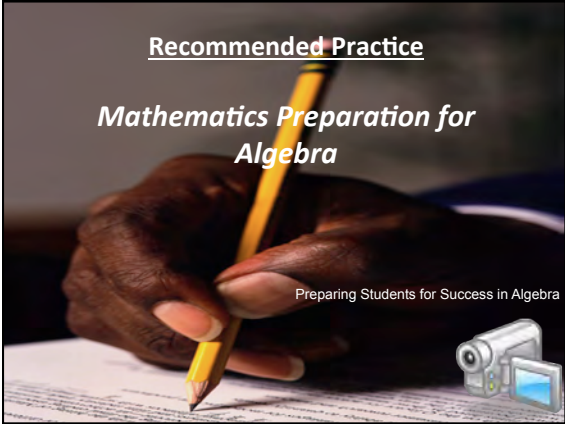






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Recommended Practice

Mathematics Preparation for Algebra




Preparing Students for Success in Algebra



Key Concepts

- Develop students' conceptual understanding and procedural knowledge of whole numbers, fractions, decimals, percents, and proportional reasoning and problem solving.
 - Classroom instruction must focus on the foundational topics of whole numbers and especially fractions. Students need to develop conceptual understanding and procedural knowledge to be successful at problem solving, and experience a mix of problem types and procedures to become proficient.



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Key Concepts



- Develop automatic recall of arithmetic facts, fluency with standard algorithms, and number sense.
 - Students must develop number sense and become fluent with whole numbers and fractions to be successful in mathematics. Teachers need to provide students with ample opportunities to develop automatic recall of facts, which provides a foundation for learning fractions, and competence with algorithms so they can solve problems quickly and efficiently.

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Key Concepts



- Build student understanding of the commutative, distributive, and associative properties and address topics of geometry and measurement that are relevant to algebra.
 - Along with developing fluency with whole numbers and fractions, students must be able to understand the commutative, distributive, and associative properties as they master the mathematical operations of addition, subtraction, multiplication, and division. Teachers should focus some part of instruction on geometry; in particular, similar triangles and measurement.

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Learn What Works

Dr. Fennell

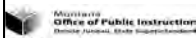
- *The Critical Foundations*
- *Benchmarks as Guideposts*
- *Professional Development for the Critical Foundations*



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Division of Assessment, Student Support and Accountability

See How it Works

- Focus on Key Topics
- Number Sense
- Fractions, Decimals, Percents



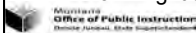
Focus on Key Topics



- MT K-12 Mathematics Content Standards

<http://www.opi.mt.gov/pdf/Standards/10MathConStds.doc>

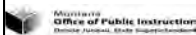
- What are Montana's key topics?
- Pacing Guides for Pre-Algebra and Algebra I
 - Describe the features of a pacing guide.
 - How might a pacing guide be useful?




Number Sense



- The term "number sense" is a relatively new one in mathematics education. It is difficult to define precisely, but broadly speaking, it refers to "a well organized conceptual framework of number information that enables a person to understand numbers and number relationships and to solve mathematical problems that are not bound by traditional algorithms" (Bobis, 1996).
- The National Council of Teachers identified five components that characterize number sense: number meaning, number relationships, number magnitude, operations involving numbers and referents for numbers and quantities. These skills are considered important because they contribute to general intuitions about numbers and lay the foundation for more advanced skills.




Number Sense



3rd Grade Word Problem Example


Differentiated Student Assignments

- How does the teacher prepare his students?
- What type of learning environment is he using in this lesson?
- How are students demonstrating their understanding?



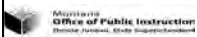
Differentiated Student Assignments

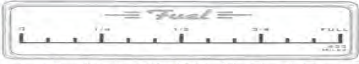
- Discussion/Write:
 - How could assignments like this be used in your classroom/school?
 - What preparation would you need to do in order for this to be successful?
 - What is the role of the teacher during an assignment like the example?
 - What might some barriers be to teachers using similar assignments?



Fractions, Decimals, Percents

- Expert Interview: Dr. Wu: Teaching Fractions
 - Reflect how you currently teach fractions and how Dr. Wu is describing the teaching of fractions.
- Using Multiple Representations to Teach Fractions
 - Keep track of those described in the video






Frank runs a business called Frank's Fresh Farm Produce. Once a week he drives north of the city to farms where he buys the best possible produce for his customers. Frank can travel 600 miles on a full tank of gas. His truck has a fancy, accurate fuel gauge.


Usually Frank has time to visit only one farm on each trip, but this week he decides to visit both Stan's and Louisa's farms. When Frank drives from his store to Stan's farm and back, he knows he uses $\frac{5}{12}$ of a tank of gas. When he drives to Louisa's farm and back, he uses $\frac{1}{3}$ of a tank of gas. From an area map, he learns that there is a road from Stan's farm to Louisa's farm that is 120 miles long. He realizes that he can drive from his store to Stan's farm, then to Louisa's farm, and then back to his store in one loop.

Frank can tell by looking at his fuel gauge that he has $\frac{5}{8}$ of a tank of gas. Can he drive this loop without having to stop for fuel? Or, should he buy gas before he starts his trip?

Using a Number Line

- Bradley Witzel from the Montana Reading Institute 2010 
- What background knowledge would a student would need to have in order to successfully solve these problems?
- What math skills would be necessary to use to solve these problems?

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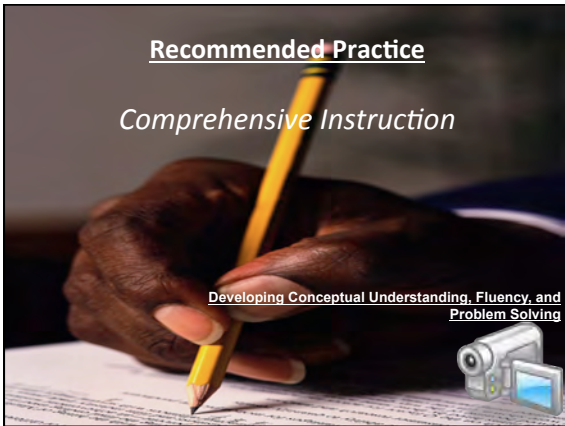



Do What Works

- Review current standards against NMP benchmarks.
- Learning Together About Mathematics Preparation for Algebra
- Planning Template #3: Working With Schools

Montana K-12 Mathematics Content Standards
<http://www.opi.mt.gov/pdf/Standards/10MathConStds.doc>

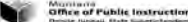
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Key Concepts


- Simultaneously develop students' conceptual understanding, procedural fluency, and problem-solving skills.
 - Mathematics instruction should focus on teaching conceptual understanding of mathematical operations, proficiency with operational procedures, and fluency with basic number facts all together, as each facilitates and reinforces learning in the others and they jointly support effective and efficient problem solving.





Key Concepts

- Provide adequate practice opportunities to develop fluency with arithmetic facts and standard algorithms.
 - Teachers should provide practice that is distributed over time and includes a conceptually rich and varied mix of problems to help students develop fluency with facts and proficiency in operations, conceptual understanding, and problem solving.



Learn What Works

• Dr. Ferrini-Mundy

– Lesson focus

- Computational Fluency
- Conceptual Understanding
- Problem-Solving

– What can schools do?

– What about student motivation and the teacher's role in student motivation?





See How it Works

• Conceptual and Procedural Understanding

• Effort and Persistence



Conceptual and Procedural Understanding



• Why is it important to develop students understanding of number sense while at the same time developing fluency with facts?



– An Administrator's Perspective on Mathematics Instruction

• How can we fit it ALL in?

– Specialist Teachers Provide Practice in Mathematics

Effort and Persistence



- It matters!
- We need to have a plan to support and build on effort, stamina, and persistency



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- What can I do to help struggling students persist when trying to solve problems?
- How can teachers reflect on our own teaching strategies to ensure that we are consistent in our communication about effort and practice?
- How can we work with other teachers in our schools to strengthen elements of the school culture that will support persistence?

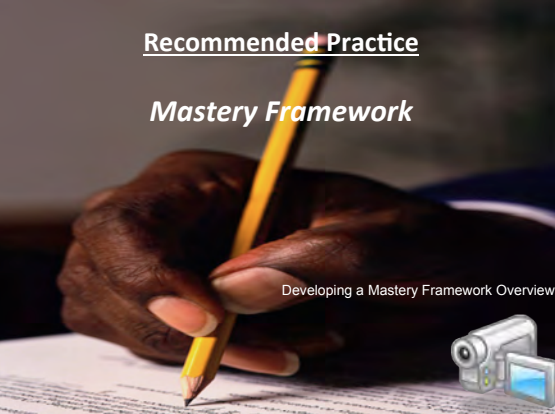
Do What Works




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Recommended Practice

Mastery Framework



Developing a Mastery Framework Overview

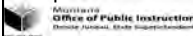


Key Concepts



- Establish benchmarks based on the Critical Foundations for Algebra to systematically gain mastery of key topics.

- District and school benchmarks, curriculum, and assessments should be aligned with the benchmarks for the critical foundations and mastery points recommended by the National Mathematics Advisory Panel to ensure that instruction focuses on essential skills and topics and student learning proceeds at an effective pace.

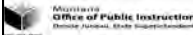


Key Concepts



Regularly use formative assessments to target and differentiate instruction.

- Teachers should use formative assessment on a regular basis to monitor progress for all students and systematically examine assessment results to determine which groups of students should receive what types of interventions and to differentiate instruction for individuals needing more help with concept and skill development. For struggling students, formative assessment should be used once or twice a week.



Key Concept



Use a combination of instructional methods for students experiencing mathematical difficulties, including explicit instruction, multiple representations, and additional materials



Regular, explicit methods of instruction help students in the performance of computations, solving word problems, and solving problems that require the application of mathematics to novel situations.

This time should be dedicated to ensuring that students possess the foundational skills and conceptual knowledge necessary for understanding the mathematics they are learning at their grade level.



Key Concepts



Differentiate instruction for mathematically gifted students.

- Schools need to support appropriately challenging work in mathematics for gifted and talented students. Gifted students need a curriculum that is differentiated (by level, complexity, breadth, and depth), developmentally appropriate, and conducted at a more rapid rate. A mathematics program that includes a combination of acceleration and enrichment components is recommended.

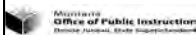





Learn What Works

Use a mastery framework to guide instructional planning and student assessment

- Expert Interview: Dr. Fuchs






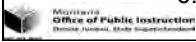
Learn What Works

Dr. Fuchs: Research-Based Instructional Programs

Design Features


1. Explicitness
2. Instructional Design
3. Conceptually Rich
4. Drill and Practice
5. Cumulative Review
6. Motivating Instruction

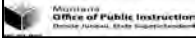




See How it Works

- Benchmarks for Mastery
- Formative Assessments
- Working with All Students:
Struggling and Accelerated





Critical Foundations for Algebra

The National Mathematics Advisory Panel report offers recommendations for how we can best prepare elementary and middle school students for success in algebra, a gateway to mathematics in high school and beyond.

Mathematics Preparation for Algebra

- Follow a focused, coherent progression of mathematics
- Achieve proficiency in computation

Benchmarks for Mastery See How it Works:

Comprehensive Instruction


- Develop conceptual understanding, computational fluency, and problem-solving skills
- Achieve automaticity in computation
- Provide adequate practice
- Encourage effort and persistence

Mastery Framework

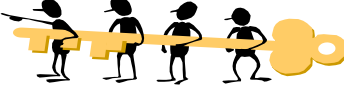
- Set benchmarks for key skills
- Use formative assessments
- Provide explicit instruction for struggling students
- Offer acceleration and enrichment for gifted students

Grade-level Benchmarks for Critical Foundations



800	● Add and subtract whole numbers
600	● Identify, represent, and compare fractions and decimals
400	● Multiply and divide whole numbers
200	● Compare, add, and subtract fractions and decimals
0	● Solve problems with perimeter and area
-200	● Multiply and divide fractions and decimals
-400	● Use all operations on integers
-600	● Analyze properties and resources with 2- and 3-D shapes
-800	● Use all operations on positive and negative fractions
-1000	● Solve problems with percent, rate, ratio, and proportion
-1200	● Measure similar triangles with slope of a line



See How it Works
.....Formative Assessments




- Reviewing student work
 - Protocols for Reviewing Student Work
- Formative Assessment on a Daily Basis
 - Six Elements of an Effective Math Lesson






See How it Works
Working with **All** Students

- Interventions for Struggling Students transcript



- Acceleration for Mathematically Gifted Students transcript


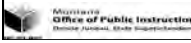




Do What Works

The Components of a Mastery Framework

- A checklist to aid staff to understand the key components of a mastery framework and to assess their school' s progress toward implementing these components.

The Components of a Mastery Framework

Discuss in your teams:

- Which components are in place?
- Which components are partially in place?
- Which components are not at all in place?
- What are our priorities?
- Who are the persons responsible?






Do What Works

- Working with Struggling Students
 - Self-Assessment Inventory
 - This tool has been designed to help teachers identify whether or not they are implementing all the practices that might help students who are struggling in mathematics.




Classroom Instruction	
1. In each lesson I demonstrate to students how to solve problems.	Step-by-step modeling Think-alouds (sharing how I address a problem) Demonstrating more than one way to address a problem Explicit directions and explanations
2. I demonstrate multiple examples of similar problems.	Examples that are similar Examples with slight variations Alternating challenging problems with easier problems
3. I use multiple representations in demonstrations.	Concrete materials Representational materials Story contexts Visual diagrams Virtual demonstrations (via computer) Number line
Instruction: What might you add to your instruction or do more frequently to provide clearer instruction to students who are struggling?	
Student Practice	
1. I encourage students to think aloud while solving problems.	Explain work to teacher Document steps taken in problem solving Using drawing or models to represent thinking Write about their work
2. I provide many opportunities for practice.	Guided practice (students work a problem and approach is checked) In-class independent practice (support available) Worked examples are included for independent practice Homework
3. I encourage students to explain their solutions to other students.	Working with a partner Working in cooperative groups Demonstrating worked solutions to class







Do What Works

- Working with Mathematically Gifted Students
 - This tool has been designed to help school mathematics leaders and teachers to reflect about how well they are implementing practices to engage and challenge students who are gifted in mathematics.

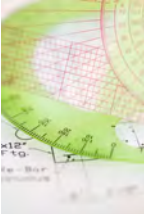







Do What Works—Planning Template


- What does your school have in place?
- What does your school need to put into implementation?
- How do we prioritize the needs and who will be involved (roles/responsibilities)?
- What is our timeline for the action plan items?



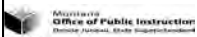


National Mathematics Advisory Panel

- “For all content areas, practice allows students to achieve automaticity of basic skills...which frees up working memory for more complex aspects of problem solving.”



- “Students learn by building on prior knowledge, extending as far back as childhood.”



References/Resources

- ❑ Doing What Works: <http://dww.ed.gov/>
- ❑ National Mathematics Advisory Panel Final Report:
[http://www2.ed.gov/about/bdscomm/list/mathpanel/
report/final-report.pdf](http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf)
- ❑ Montana Office of Public Instruction RtI
[http://opi.mt.gov/Resources/RTI/
Index.html#gpm1_2](http://opi.mt.gov/Resources/RTI/Index.html#gpm1_2)
- ❑ Montana Office of Public Instruction Content
Standards: [http://www.opi.mt.gov/Curriculum/
Index.html](http://www.opi.mt.gov/Curriculum/Index.html)

